COURSE CONTENT

Academic Year	:	2013/2014, Semester II.
Course Code & Title	:	Quantum Probability Undergraduate students register under: MAS463/MTH473/MH4932 Special Topics in Statistics Graduate students register under: MAS 723 Topics in Probability and Statistics I.
To apply	:	Please request and submit a form from the MAS general office.
Prerequisites	:	Permission of the lecturer, but an informal list of prerequisites would include MH2500, MH3100, MH3200. Students are encouraged to take MH4100 at the same time.

Lecturer

Ian Flint, with the participation of Uwe Franz (visiting) and Nicolas Privault

Track requirements

Undergraduate students of any track can use this course to contribute to their track requirements. The division will view it as being on all lists.

Who is this an appropriate course for?

This will be a challenging course which should be of interest to theoretically-minded students with an interest in any of the following topics: advanced probability, quantum physics, real analysis and measure theory, and abstract algebra.

Timetable

L: Wed 0830 - 1030

L: Fri 1530 - 1630

T: Fri 1630 - 1730

Course outline:

Quantum probability is an algebraic approach to probability theory in which operators are viewed as non-commuting random variables whose probability distributions are given by their spectral measures. This field has deep connections with quantum physics, for example in the basic quantum harmonic oscillator model the pair (p,q) of non-commuting position and momentum observables satisfy the canonical commutation relation [p,q] = I and can be seen as a two-dimensional non-commutative random variable with Gaussian marginal distributions. Due to the use of non-commutative methods in probability, this topic is also connected to other fields in statistics such as random matrices.

This course will broadly cover the algebraic and probabilistic aspects of quantum probability while emphasizing some applications to quantum optics. It will also include a review of the necessarily algebraic prerequisites on real Lie algebras, as well as some required background on classical probability. Other related topics of current research interest, such as determinantal point processes, will be also considered.

Learning Objective: To master the algebraic and probabilistic tools of quantum probability as a preparation for research in domains involving non-commutativity and statistical/physical applications.

Content

I. Real Lie Algebras

I.1 The Heisenberg-Weyl Lie algebra HW
I.2 The Oscillator Lie algebra OSC
I.3 The Lie algebra sl(2,R)

II. Non-Commutative Random Variables

II.1 Non-commutative probability spaces
II.2 Example: the fundamental noises on the boson Fock space
II.3 Gaussian and Poisson random variables on OSC
II.4 Meixner, Gamma, and Pascal random variables on sl(2,R)

III Functional calculus on Lie algebras

III.1 Functional calculus
III.2 Wigner functions for P and Q
III.3 Generalized Wigner functions

IV The Malliavin calculus

IV.1 Fock space
IV.2 Analysis on the Wiener space
IV.3 Brownian motion
IV.4 Annihilation and creation operators

V. The non-commutative Wiener space

V.1 Derivation operator
V.2 Divergence operator
V.3 Relation to the commutative case
V.4 Sufficient conditions for the existence of smooth densities

VI. The case of sl(2,R)

VI.1 Malliavin calculus on the affine algebra
VI.2 Relation to the commutative case

References/Textbook:

Typeset notes including detailed references will be handed out for most lectures.