Detailed Answers

Mechanics

1. Correct answer D (40.6% of cohort). As the Newton (N) is equivalent to kg ms^{-2}, N s^2 kg^{-1} is equivalent to the metre (m), which is the SI unit of length. The unit of length employed by scientists working on nanotechnology is the nanometer, given by nm. The light year is the length commonly used by astronomer to indicate the distance that light traveled in 1 year.

   Choice A (13.2% of cohort). The students may have expected the unit of length to be explicit, but it can be implicit like N s^2 kg^{-1}, which is equivalent to the metre (m). The students may have mistaken the light year to be a unit of time.

   Choice B (23.3% of cohort). See choice A above.

   Choice C (22.6% of cohort). See choice A above.

   Choice E (0.3% of cohort). The students have mistaken the tesla to be a unit of length. It is in fact the derived SI unit of the magnetic flux density. In terms of the SI based unit, 1 tesla is equivalent to 1 kg A^{-1} s^{-2}.

2. Correct answer B (60.0% of cohort). Systematic errors are consequence of biasness in measurement. The average of such measurement will differ significantly from the actual value of the quantity to be measured. Since such errors can result from the drift in certain aspects of the components of an instrument, re-calibrating the instruments will rectify such errors. On the other hand, random errors are caused by the unavoidable fluctuations in the readings of an instrument or the interpretations of the experimenter on the instrument's reading. The latter situation occurs on reading the scale of the ruler during length measurement.

   Choice A (18.7% of cohort). The students may have perceived that length measurement through scale reading only leads to systematic errors. This is not true. Systematic errors will have occurred only if the rulers have become distorted.

   Choice C (7.8% of cohort). The students do not have the correct concept that random errors can be reduced through an averaging of a large number of results.

   Choice D (2.4% of cohort). Same as choice C above.

   Choice E (10.8% of cohort). The average of the four measured results are: (57.46 g + 57.49 g + 57.45 g + 57.47 g)/4 = 57.47 g, which gives a bias when compared to the actual mass of 56.50 g. Hence, there are systematic errors in addition to random errors in the measurements.

3. Correct answer D (73.3% of cohort). A scalar is a quantity that possesses magnitude but not direction. Physical quantities such as mass, voltage and temperature are scalars.

Choice A (3.2% of cohort). The students have mistaken that the value of a scalar must be positive. This is not true. The value of a scalar can be negative, like the subzero temperature of -12 °C.

Choice B (19.9% of cohort). The students have not considered the voltage as a scalar quantity. Voltage is electric potential energy per unit charge. Hence, like energy, it is also a scalar.

Choice C (0.9% of cohort). The students have confused scalars with vectors. 7**i** + 2**j** and displacement have both magnitude and direction. They are vectors and not scalars.

Choice E (2.8% of cohort). The students have probably confused displacement with distance. The former is a vector, while the latter is a scalar.

4. Correct answer B (23.9% of cohort). The vertical displacement s covered in time t is

$$s = ut + \tfrac{1}{2}gt^2$$

where g is the acceleration due to gravity and u is the initial vertical velocity. Since s, g and t are all the same for the two bullets, u must be the same. The vertical velocity and hence the vertical speed of the bullets are equivalent.

Choice A (23.1% of cohort). The horizontal displacement d covered in time t is

$$d = vt$$

where v is the initial horizontal velocity. While t is the same for both bullets, the two guns may differ in their horizontal location, resulting in different horizontal displacement. Thus, horizontal speed may be different as well.

Choice C (11.6% of cohort). As the initial horizontal velocity v may differ while the initial vertical velocity u is the same for the two bullets, the guns may be fired at different angles to the horizontal.

Choice D (5.0% of cohort). Since horizontal displacement d may be different while the vertical displacement s must be the same for the two bullets, the distance between the target and the guns may be different.

Choice E (35.8% of cohort). Maximum height is reached at time T when the vertical velocity is zero:

$$0 = u + gT$$

As the initial velocity u (cf. answer to Choice B) and acceleration due to gravity g are the same for the two bullets, the time to reach maximum height must be the same. Hence, the maximum height obtained is also the same given the same initial height and vertical velocity.

5. Correct answer C (68.3% of cohort). If we consider the air resistance, during the falling process, the magnitude of the acceleration decreases.

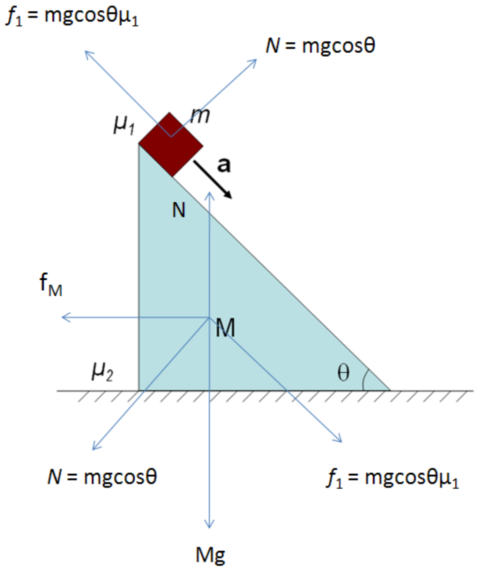
   Choice A (3.2% of cohort). This statement is true. The student chooses this answer if he/she forget to consider the air resistance.

   Choice B (7.7% of cohort). This statement is true. The student may be confused about the concepts of speed and acceleration.

   Choice D (13.3% of cohort). This statement is true. Air resistance creates a lesser effect if the ball is heavier.

   Choice E (7.6% of cohort). This statement is true. Air resistance always reduces the acceleration.

6. Correct answer C (21.7% of cohort).



Method I: Consider m and M as one system.
The net horizontal acceleration force is $ma\cos\theta$ which is supplied by the friction force between the ground and M.
The net vertical acceleration force is $ma\sin\theta$ which is the difference between $(M+m)g$ and normal force N.
So, $\underline{f = ma\cos\theta}$ and $N = (M+m)g - ma\sin\theta$.

Method II: Decompose the forces between m and M (see figure above).

$f_M = mg\cos\theta\mu_1\cos\theta - \text{mg } \cos\theta \sin\theta$

$\underline{N = mg \cos\theta\mu_1 \sin\theta + mg \cos\theta \cos\theta + Mg}$

Choice A (22.3% of cohort). The student may have wrongly understood that the expression of the normal force is $(M+m)g$ on M.

Choice B (13.2% of cohort). The student may have missed μ_1 in the expression of f and mistaken $\cos\theta$ for $\sin\theta$ in the expression of N.

Choice D (34.2% of cohort). The expression of f is the friction between m and M. The student may have wrongly understood that the expression of the normal force is $(M+m)g$ on M.

Choice E (6.0% of cohort). Wrong guess.

7. Correct answer E (69.8% of cohort). Q is acted upon by two forces: the tension T_Y in string Y which acts upwards, and the weight W_Q acting downwards. The net force acting on Q is the difference between the two, and this has to accelerate Q upwards at 2 m/s^2. T_Y must therefore be $W_Q + ma$, which is 22 N. P is acted upon by three forces: the tension T_Y in string Y acting downwards, the tension T_X in string X acting upwards, and the weight W_P acting downwards. The net force acting on P is therefore $T_X - T_Y - W_P$, and this must produce an acceleration of 2 m/s^2 upwards. Hence $T_X = T_Y + W_P + ma = 22 + 1.7 + 0.3 = 24$ N.

Choice A (4.1% of cohort). The student just makes a guess.

Choice B (6.0% of cohort). The student just makes a guess.

Choice C (8.7% of cohort). The student just makes a guess.

Choice D (10.5% of cohort). The student just makes a guess.

8. Correct answer C (10.6% of cohort). Since the Earth, the body, and the spring is a closed system, total linear momentum must be conserved when there is no external force acting. When the body comes instantaneous at rest, its original momentum must have been transferred to the Earth via the spring.

Choice A (76.6% of cohort). *Energy* has been stored in the spring, not momentum.

Choice B (4.2% of cohort). Momentum cannot be transferred to the body itself.

Choice D (1.2% of cohort). This is a perfectly elastic collision, thus energy is conserved.

Choice E (7.2% of cohort). The collision is perfectly elastic.

9. Correct answer B (39.6% of cohort). The force F exerted on the ball by the table during the collision must accelerate the ball in the upwards direction. Otherwise the ball cannot rebound. Since the ball is always acted upon by its weight mg downwards, F must be larger than mg for the ball to rebound.

Choice A (17.9% of cohort). If this were true, the ball would not have rebounded upwards.

Choice C (16.3% of cohort). Violates Newton's 3rd Law --- F must be equal to the force that the ball exerted on the table during the collision

Choice D (24.0% of cohort). Violates Newton's 3rd Law --- F must be equal to the force that the ball exerted on the table during the collision

Choice E (2.0% of cohort). Students will choose this if they somehow think that the ball is instantaneously stationary during the collision.

10. Correct answer C (65.5% of cohort). The time that elapses until the cars crash is $\frac{D}{2V_1}$. The fly will travel a distance of $V_2 \frac{D}{2V_1}$.

Choice A (4.5% of cohort). The students may have forgotten to include the movement of the second car and thought only the first car is moving.

Choice B (6.0% of cohort). Random answer.

Choice D (6.2% of cohort). The students may have thought V_2 is measured relative to the car and that only the first car is moving.

Choice E (16.0% of cohort). Almost correct, but the students may have thought that V_2 is measured relative to the car.

11. Correct answer C (4.7% of cohort). The van der Waals interaction between neutral atoms can be explained by the electrostatic interaction between instantaneous dipoles induced by each other.

Choice A (29.6% of cohort). The student has erroneously assumed that the electromagnetic force acts only between objects with non-zero net charges. The student has erroneously assumed that electrostatic and gravitational forces can only go as $1/r^2$.

Choice B (24.0% of cohort). The student has erroneously assumed that the electromagnetic force acts only between objects with non-zero net charges. The student has erroneously assumed that electrostatic and gravitational forces can only go as $1/r^2$.

Choice D (26.5% of cohort). The student has erroneously assumed that the electromagnetic force acts only between objects with non-zero net charges. The student has erroneously assumed that electrically neutral objects can only interact gravitationally.

Choice E (15.0% of cohort). The student has erroneously assumed that the electromagnetic force acts only between objects with non-zero net charges. The student has erroneously assumed that electrostatic and gravitational forces can only go as $1/r^2$.

12. Correct answer C (14.1% of cohort). The upper portion of the spring has to support a larger weight (load + weight of lower portion of spring) compared to the lower portion of the spring (load only), and thus the extension in the upper portion of the spring should be larger than the extension in the lower portion of the spring.

Choice A (7.7% of cohort). Students will probably not choose A, as it looks as if the spring is not stretched.

Choice B (45.8% of cohort). The student has neglected the weight of the spring.

Choice D (27.0% of cohort). The student realized that the mass of the spring will cause different parts of the spring to extend differently, but got the logic in C wrong.

Choice E (5.1% of cohort). The student realized that the mass of the spring will cause different parts of the spring to extend differently, but thought that since the spring is pulled on both ends, it is the centre that gets extended the most.

13. Correct answer C (25.3% of cohort). In order for the boat to float on water the upthrust (or buoyant force) by the water to the boat due to the fact that the boat displaces some water must be equal to the weight of the boat.

Choice A (11.3% of cohort). The boat can be made by materials with higher density than seawater.

Choice B (9.4% of cohort). There are boats of different volumes.

Choice D (47.9% of cohort). When the boat floats on water the upthrust (buoyant force) by the water to the boat is equal to the weight of the boat.

Choice E (6.1% of cohort). The student makes a guess.

14. Correct answer E (47.7% of cohort). The increase in kinetic energy is given by

$$\Delta E = \frac{1}{2} m a^2 [(t+\Delta t)^2 - t^2] = \frac{1}{2} m a^2 (2t\Delta t + \Delta t^2)$$

$$\frac{\Delta E}{\Delta t} = m a^2 t + \frac{1}{2}\Delta t$$

where *a* is the constant acceleration, *m* the mass, *t* the time and *Δt* the infinitesimal time interval. Therefore the energy output increases with time and the rate of increase ma^2 is a constant.

Choice A (23.3% of cohort). Students may think that a constant power output can sustain a constant acceleration.

Choice B (5.0% of cohort). Students just make a wild guess.

Choice C (7.1% of cohort). Students just make a wild guess.

Choice D (16.5% of cohort). Students know the idea but do not know how to derive the formula.

15. Correct answer C (55.4% of cohort). By the principle of conservation of energy, we have for either the golf ball or the ping pong:

$$mgh + \mu mgs = \frac{1}{2} m v^2$$

$$h = \frac{v^2}{2g} - \mu s$$

where m is the mass, h is the vertical height up the slope, μ the coefficient of kinetic friction, s the total distance traveled, v is the initial velocity, while g is the gravitational constant. With the final expression of h being independent of mass, it is easy to deduce that both the golf ball and ping pong ball eventually reach the same height.

Choice A (25.7% of cohort). Students may think that the larger kinetic energy will eventually be transferred to a larger potential energy of the ball up the slope. As a result, conclude that the ball will get to a higher position.

Choice B (5.1% of cohort). Students may think that the smaller frictional force on the ping-pong ball has resulted in its reaching a greater height.

Choice D (8.8% of cohort). Students have made a wild guess.

Choice E (4.5% of cohort). Students have made a wild guess.

16. Correct answer E (27.9% of cohort). During inelastic collision, momentum is conserved, but kinetic energy is not conserved. The kinetic energy can decrease after the collision, where the energy is lost as heat and sound. Alternatively, the kinetic energy can increase after the collision, as happen after an explosion.

Choice A (3.7% of cohort). This is the definition of elastic collision, not inelastic collisions.

Choice B (3.7% of cohort). Students have the wrong concept that momentum is not conserved during an inelastic collisions.

Choice C (2.4% of cohort). Students do not have any understanding on the concept of elastic and inelastic collision.

Choice D (62.3% of cohort). Students do not know that the kinetic energy can increase after an inelastic collision, such as an explosion.

17. Correct answer C (59.8% of cohort). An angular velocity of N rpm is equivalent to a rotational frequency of $N/60$ Hz.

Choice A (6.2% of cohort). Students may have forgotten the difference between Hz and rpm.

Choice B (7.6% of cohort). Students have mixed up the concepts of revolution and radian.

Choice D (25.1% of cohort). See choice B.

Choice E (1.0% of cohort). Students are confused with the concepts of angular velocity and tangential velocity.

18. Correct answer C (40.5% of cohort). At the lowest point of the swing, the acceleration the boy experiences is the centripetal acceleration pointing vertically upwards, which is v^2/l. At the highest point, the acceleration points along the tangential direction (that is $g \sin \theta$) since there is no centripetal acceleration because the speed of the boy is zero at this point.

Choice A (16.4% of cohort). Students have mistaken that the boy has become weightless at the highest point.

Choice B (11.3% of cohort). Students have forgotten to consider the tangential component of g which is the resultant acceleration at the highest point.

Choice D (18.5% of cohort). Students have forgotten that at the lowest point, the gravitational force acting on the boy has been cancelled by the tension from the swing.

Choice E (12.0% of cohort). See choice B and D.

19. Correct answer B (67.0% of cohort). While the satellite orbits about the Earth, the centripetal force acting on it is equal to the gravitational force. Therefore, $v^2/r = GM_{Earth}/r^2$, where $r = R_{Earth} + h$. This gives $v = (GM_{Earth}/r)^{1/2}$.

Choice A (9.6% of cohort). Students have not taken the height h of the satellite into account.

Choice C (9.8% of cohort). Students may be confused between the speed of the satellite and the radius of the orbital.

Choice D (5.8% of cohort). Students have made errors during the calculations.

Choice E (5.8% of cohort). Students may have used GM_{Earth}/r as the formula for the gravitational force – leading to $v = (GM_{Earth})^{1/2}$.

20. Correct Answer E (84.3% of cohort).

The gravitational acceleration $g = \frac{GM_E}{r_E^2}$, therefore, the ratio $\frac{g}{G} = \frac{M_E}{r_E^2}$

Choice A (2.6% of cohort). These are incorrect derivations. The student probably does not understand the fundamental of gravitational acceleration.

Choice B (9.6% of cohort). These are incorrect derivations. The student probably does not understand the fundamental of gravitational acceleration.

Choice C (1.2% of cohort). These are incorrect derivations. The student probably does not understand the fundamental of gravitational acceleration.

Choice D (1.9% of cohort). These are incorrect derivations. The student probably does not understand the fundamental of gravitational acceleration.

21. Correct Answer B (62.3% of cohort). The space station is not free from the Earth's gravity. It is the Earth's gravity that keeps the space station, and the astronaut inside it, in orbit around the Earth. The feeling of "weightless" is because the space station and all things inside it are in free fall and both have the same acceleration. The ISS and the astronaut inside it are moving with a velocity that is in perpendicular with the Earth's gravity, and is high enough to prevent them crashing to the Earth. As they fall towards the Earth, the surface of the Earth curves away from them.

Choice A (12.1% of cohort). This is a false statement because the space station and everything inside it require the Earth's gravity to keep them continued rotating the Earth.

Choice C (25.1% of cohort). This is false statement because the space station and the astronaut are constantly in free fall and both have the similar acceleration.

22. Correct Answer C (60.0% of cohort).
When the spaceship just escapes from the asteroid (i.e it is infinitely far from the asteroid) the final potential and kinetic energies are zero. Conservation of energy yields

$$-\frac{GMm}{r}+\frac{1}{2}mv^2=0$$

We replace *GM/r* with *gr*, where *g* is the gravitational acceleration at the asteroid surface. Then, the energy equation is $-gr + ½v^2 = 0$. We solve for the escape velocity *v*:

$$v=\sqrt{2gr}=\sqrt{2\left(3.0\text{m/s}^2\right)\left(300\times10^3\,\text{m}\right)}=1.3\times10^3\,\text{m/s}.$$

Choice A (5.9% of cohort). Wild guess. The student probably does not understand the concept of escape velocity.

Choice B (20.8% of cohort). Wild guess. The student probably does not understand the concept of escape velocity.

Choice D (7.9% of cohort). Wild guess. The student probably does not understand the concept of escape velocity.

Choice E (3.5% of cohort). Wild guess. The student probably does not understand the concept of escape velocity.

23. Correct Answer B (36.7% of cohort).

The gravitational acceleration is given by $g = GM_E / r_E^2$, where M_E is the mass of Earth and r_E is the Earth's radius. We substitute $r = r_E + h$, where h is the distance from the Earth's surface. Then, $g = GM_E / \left(r_E + h\right)^2$.

Solving for the distance, $h = \left(\sqrt{GM/g}\right) - r_E$. Substituting the values, $r_E = 6.37 \times 10^6$ m and $M_E = 5.98 \times 10^{24}$ kg, we obtain

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\mathrm{m}^3/\mathrm{s}^2\cdot\mathrm{kg}\right)\left(5.98\times10^{24}\mathrm{kg}\right)}{\left(3.3\mathrm{m}/\mathrm{s}^2\right)}} - 6.37\times10^6\mathrm{m} = 4.6\times10^6\mathrm{m}.$$

Choice A (3.2% of cohort). Wild guess. The student probably did not do the calculation correctly or he/she does not understand the concept of gravitational acceleration.

Choice C (41.1% of cohort). Wild guess. The student probably did not do the calculation correctly or he/she does not understand the concept of gravitational acceleration.

Choice D (12.9% of cohort). Wild guess. The student probably did not do the calculation correctly or he/she does not understand the concept of gravitational acceleration.

Choice E (5.8% of cohort). Wild guess. The student probably did not do the calculation correctly or he/she does not understand the concept of gravitational acceleration.

24. Correct answer A (31.9% of cohort). Since this is an isolated and conservative system, we expect the conservation of total energy. The conservation of total linear momentum also applies since there is no external force. Although there is mutual gravitational interaction between the two planets, the two forces are equal and opposite. Note that these two forces are known as the internal forces.

By applying the conservation of total energy:

$$-\frac{Gm_1m_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + -\frac{Gm_1m_2}{d/2},$$

and the conservation of total linear momentum:

$$m_1v_1 = m_2v_2,$$

and then solving for v_1 and v_2, the answers given in choice A is obtained

Choice B (22.0% of cohort). The students have not perceived the need to apply the principle of conservation of linear momentum when there is an absence of external forces acting on the system. Furthermore, the students have erroneously assumed $m_1 = m_2$ to solve the problem, and then put back the masses arbitrarily in the final solution.

Choice C (26.9% of cohort). The students have again not considered the principle of conservation of total linear momentum. In addition, they have made the wrong assumption that the velocity of the two masses are the same.

Choice D (13.3% of cohort). As in choice C, but the students have made the further error of taking the gravitational potential to be $-Gm_1m_2/d^2$. The correct gravitational potential to be employed is $-Gm_1m_2/d$.

Choice E (1.9% of cohort). The students have made further conceptual errors beyond those discussed above.

Thermodynamics and Matter

25. Correct answer B (83.3% of cohort). By $\Delta Q = mc\,\Delta T$, where ΔQ is the amount of heat absorbed by the substance, m is its mass, c is its specific heat capacity, and ΔT its temperature change, we observe that ΔT is inversely proportional to c if m and ΔQ are constant. This implies that $\Delta T_{water} < \Delta T_{iron}$ since $c_{water} > c_{iron}$.

Choice A (0.6% of cohort). Students have misconceived that the same amount of added heat will induce the same change in temperature.

Choice C (3.5% of cohort). Students has assumed that ΔT is proportional to c if m and ΔQ are constant.

Choice D (10.3% of cohort). Students did not understand the concept of specific heat capacity.

Choice E (2.3% of cohort). Same as choice D.

26. Correct answer B (40.3% of cohort). The amount of heat removed by cooling 100g of water from 6 ºC to 0 ºC is equivalent to the heat of fusion absorbed to melt x g of ice. Therefore, one can obtain the answer by solving: 333×10^3 J/kg $\times$ $x/1000$ kg = 0.1kg $\times (6 - 0) \times$ 4190 J/kg.K.

Choice A (1.4% of cohort). The students have made a random guess.

Choice C (3.0% of cohort). Students have ignored the latent and specific heats, and have simply used the numbers given to obtain: 100×6/50 = 12.

Choice D (18.1% of cohort). Students may have selected this choice through their daily experience in coffee shops where ice cubes in their drinks will eventually melt. This is so because heat from the surrounding is allowed to enter the cup, but for this question, the cup is insulated.

Choice E (36.5% of cohort). Students have failed to quantitatively relate the concepts of specific heat and latent heat.

27. No answer.

28. Correct answer is D (21.1% of cohort). This question tests students' understanding of the first law of thermodynamics. In the process (2), there is no heat exchange and so the work done by the surroundings lead to an increase of temperature in the gas.

Choice A (12.8% of cohort). The student may think wrongly that the gas does work, so the temperature should decrease.

Choice B (11.4% of cohort). The student may think wrongly that the temperature changes after the cycle completes.

Choice C (20.8% of cohort). The student makes a mistake about the sign of the net work done.

Choice E (33.5% of cohort). The correct answer is D.

29. Correct answer B (89.6% of cohort). The heat absorbed by the water should be equal to the heat released from the metal, or

$$0.2kg \times 4200Jkg^{-1}K^{-1} \times (40^0C - 20^0C) = 0.2kg \times x \times (200^0C - 20^0C)$$

which gives x=525 J kg^{-1}K^{-1}.

Choice A (2.5% of cohort). The student makes a guess.

Choice C (2.8% of cohort). The student makes a guess.

Choice D (1.4% of cohort). The student makes a guess.

Choice E (3.5% of cohort). The student makes a guess.

Waves and Oscillations

30. Correct answer B (58.7% of cohort). The period of oscillation can be determined through the angular frequency $\omega = (k/m)^{1/2}$ where m is the mass. Thus, the period of oscillation is $T = 2\pi/\omega$. The initial displacement is not changing the period of oscillation. Inserting the constants gives $T = 1.77$ s, or frequency $f = 0.56$ Hz

    Choice A (8.2% of cohort). The student makes a guess.

    Choice C (18.4% of cohort). The student makes a guess.

    Choice D (8.7% of cohort). The student makes a guess.

    Choice E (2.5% of cohort). The student makes a guess.

31. Correct answer A (59.6% of cohort). The mass undergoes harmonic oscillations, say $\Theta = A \sin(\omega t)$ with the angle $\Theta = 0$ at $t = 0$. The angular acceleration is $\alpha = -A \omega^2 \sin(\omega t)$. The magnitude of the angular acceleration is therefore largest at maximum deflection.

    Choice B (16.1% of cohort). There the velocity is highest. The student mixes up velocity and acceleration.

    Choice C (7.4% of cohort). The student makes a guess.

    Choice D (12.8% of cohort). The student has not understood that for an oscillation to happen, the acceleration has to vary and change sign. A constant acceleration would lead to continuous increasing amplitude.

    Choice E (3.8% of cohort). A guess from the student

32. Correct answer E (39.1% of cohort). The solution for the harmonic oscillator is a sum of cos- and sin-function with the same argument, i.e. ωt. This sum can be written as a single sine or cosine with argument ωt and a phase factor φ. The phase angle is determined from the initial condition of the oscillation. As there is no specific initial condition mentioned, the phase angle not variable. A sin- and cos-function differ only by phase difference of $\pi/2$ in the argument. Thus, there is only a difference in the phase angle from a solution using sine or cosine.

    Choice A (15.1% of cohort). (i) is the typical presentation in a text book together with answer (iii), yet it is not the only presentation of the solution.

    Choice B (23.3% of cohort). (i) and (ii) are similar, the student may think that cosine but not a sine is a solution.

Choice C (14.8% of cohort). (iii) is a typical presentation in text books, too; yet, it is not the only presentation of the solution.

Choice D (6.5% of cohort). (iii) and (iv) are similar, the student may think that only sine are solutions but not cosine.

33. Correct answer E (82.6% of cohort). The phase speed, *c,* wavelength λ, and frequency *f,* are related through $c=\lambda f$. The speed is 1 cm/ms=10 m/s, the frequency is 10^{-3} Hz, thus the wavelength is 10^{4} m. There is no difficulty in this question except to take care of the units cm/ms and mHz.

Choice A (2.7% of cohort). This is a wild guess.

Choice B (4.4% of cohort). The student may have missed the "ms" in the denominator of the speed.

Choice C (4.1% of cohort). This is a wild guess.

Choice D (6.0% of cohort). This is a wild guess.

34. Correct answer A (32.1% of cohort). The Doppler Effect explains the change of frequency for a moving object with respect to a fixed observer. An approaching object leads to a shorter wavelength or high frequency and vice versa. Thus the wavelength of a star moving away from you is perceived with a longer wavelength. A longer wavelength is equivalent to a shift of the emission spectrum towards the red.

Choice B (16.9% of cohort). The student knows that the wavelength changes, yet assumed wrongly a shortening of the wavelength for a removing emitter.

Choice C (36.5% of cohort). The student misses the knowledge of the Doppler Effect.

Choice D (7.1% of cohort). The student has some knowledge that the wavelength may change, yet is not sure that this is a general concept and applies it to only a certain range of the spectrum.

Choice E (5.9% of cohort). Same as D or a wild guess.

35. Correct answer E (64.2% of cohort). The concept that longitudinal and transversal waves are solutions to the wave equations is tested in this question. The speed of propagation and the frequency are connected through the wavelength. The product of wavelength and frequency is not a specific property of the type of wave but of the medium.

Choice A (3.8% of cohort). This is a wild guess

Choice B (1.5% of cohort). This is a wild guess.

Choice C (20.1% of cohort). The student misses the concept of a wave, i.e. that the wave property is propagating as a function f(x+c*t) where c= λf. Therefore, we would need to know both the wavelength and the frequency.

Choice D (9.9% of cohort). This is wrong. Many mediums allow the propagation of both types of waves. Yet, mediums presented in the class such as sound in air or EM-waves allow only for either type of the wave. The student is tested to use the concept rather than the experience from class or environment.

36. Correct answer C (26.6% of cohort). This can be verified by accounting for the phase shift under reflections and Snell's law.

Choice A (10.6% of cohort). The student makes a guess.

Choice B (27.9% of cohort). The student makes a guess.

Choice D (14.7% of cohort). The student makes a guess.

Choice E (15.8% of cohort). The student makes a guess.

37. Correct answer A (38.4% of cohort). The total electric field can be described by simply adding the two time dependent vectors.

Choice B (16.4% of cohort). The student may think that the oscillating term will be averaged as it would be the case by measuring with a standard photodiode.

Choice C (16.9% of cohort). The vectors are orthogonal, meaning there is no interference term.

Choice D (11.1% of cohort). The vectors are orthogonal and additionally have no phase shift, meaning the field can not cancel.

Choice E (15.5% of cohort). This answer was just a guess from the student.

38. Correct answer C (36.9% of cohort). The intensity distribution is given by $I = I_0 \frac{\sin^2 \varphi}{\varphi^2}$ with $\varphi = (\frac{\pi d}{\lambda}) \sin \alpha$. From here one has to calculate the extrema which leads to the correct value of the intensity minima.

Choice A (4.9% of cohort). The student did not understand the basic idea of diffraction at a slit which leads to diffraction pattern behind the slit.

Choice B (32.6% of cohort). The student makes a guess.

Choice D (6.7% of cohort). The student makes a guess.

Choice E (17.3% of cohort). The student makes a guess.

Electricity and Magnetism

39. Correct answer E (36.3% of cohort). Gravitational attraction between the two asteroids of mass m each is

$$F_g = \frac{Gm^2}{r^2}$$

Electrostatic repulsion between the two asteroids of charge q each is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

For the asteroids to be in static equilibrium regardless of their distance of separation r,

$$F_e - F_g = \left(\frac{q^2}{4\pi\varepsilon_0} - Gm^2\right)\frac{1}{r^2} = 0 \quad \text{for all } r$$

$$\Rightarrow \quad \frac{q^2}{4\pi\varepsilon_0} - Gm^2 = 0$$

$$\Rightarrow \quad \frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

Thus, condition (i) is necessary.

The above expression for F_e is only valid for point charges. If we consider asteroids with significant sizes, induction would lead to uneven distributions of charges on them, hence, different expression for F_e. Therefore, condition (iii) is necessary and condition (ii) is not sufficient.

Choice A (15.1% of cohort). Condition (iii) is also necessary (see answer to Choice E).

Choice B (8.0% of cohort). Condition (i) and (iii) are necessary (see answer to Choice E).

Choice C (13.2% of cohort). Condition (i) is also necessary (see answer to Choice E).

Choice D (27.1% of cohort). Condition (iii) is also necessary (see answer to Choice E).

40. Correct answer B (25.6% of cohort). The electric field **E**(x) and potential field $V(x)$ are related by

$$\mathbf{E} = -\frac{dV}{dx}\mathbf{i}$$

Substituting $V = -2x$ obtains the correct expression for $\mathbf{E} = 2\mathbf{i}$, where ***i*** is the unit vector in the positive x-axis.

Choice A (30.9% of cohort). Substituting $V = 2x + c$, where c is a constant, yields $\mathbf{E} = -2\mathbf{i}$, which is not the given electric field.

Choice C (22.3% of cohort). Substituting $V = x^2 + c$, where c is a constant, yields $\mathbf{E} = -2x\mathbf{i}$, which is not the given electric field.

Choice D (9.6% of cohort). Substituting $V = -x^2$ yields $\mathbf{E} = 2x\mathbf{i}$, which is not the given electric field.

Choice E (10.4% of cohort). Substituting $V = 0$ yields $\mathbf{E} = 0\mathbf{i}$, which is not the given electric field.

41. Correct answer C (33.0% of cohort). If the charge is evenly distributed on each coin, the force between the two coins of charge q each at separation r is:

$$F = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \left(\frac{q}{r}\right)^2 = 4\pi\varepsilon_0 F$$

$$\Rightarrow \quad q = r\sqrt{4\pi\varepsilon_0 F} = 3.0 \times \sqrt{\frac{0.4}{9\times10^9}} = 2.0\times10^{-5}C$$

Thus, the charge on each coin is 20 μC.

However, when the coins approach one another induction leads to uneven charge distribution on the two coins, which reduces the Coulomb force between the two coins. Therefore, the charge on each coin must be >20 μC to produce the Coulomb's force between the two coins of 0.40 N.

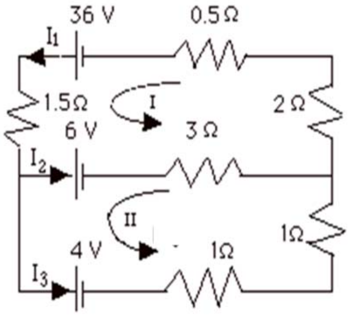
Choice A (24.3% of cohort). The student makes a guess.

Choice B (28.4% of cohort). The student think that charges are evenly distributed on each coin.

Choice D (3.5% of cohort). The student makes a guess.

Choice E (10.5% of cohort). The correct answer is Choice C.

42. Correct answer D (40.5% of cohort).



From conservation of charge,
$I_1 = I_2 + I_3$ ***or*** $I_3 = I_1 - I_2$ ***(Equation 1)***
For loop **I**,
$36\ \text{V} - I_1(0.5 + 2 + 1.5)\ \Omega - 6\ \text{V} - I_2(3\ \Omega) = 0$ *or*
$30\ \text{A} = 4I_1 + 3I_2$ ***(Equation 2)***
For loop **II**,
$6\ \text{V} + (3\ \Omega)I_2 - 4\ \text{V} - (2\ \Omega)I_3 = 0$ *or*
$2\text{A} = 2I_3 - 3I_2$ ***(Equation 3)***
Substitute Eq. (1) into Eq. (3):
$2\text{A} = 2(I_1 - I_2) - 3I_2$ ***or*** $2\text{A} = 2I_1 - 5I_2$ ***(Equation 4)***
2 x Eq. 4 equals: $4\text{A} = 4I_1 - 10I_2$ ***(Equation 5)***

Eq. 2 - Eq. 5 equals: $26\text{A} = 13I_2$ ***or*** $I_2 = 2\text{A}$.

Then from Eq. 2: $30\text{A} = 4I_1 + 3(2\text{A})$ ***and*** $I_1 = 6\ \text{A}$.

From Eq. 1, $I_3 = 6\text{A} - 2\text{A} = 4\text{A}$.

Choice A (4.3% of cohort). The student did not realize that I_2 is opposite to the direction of loop II (see figure above).

Choice B (8.7% of cohort). This is the value of I_2, not I_1.

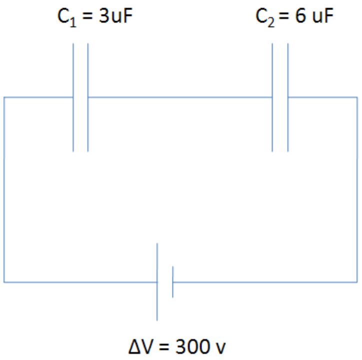
Choice C (24.3% of cohort). This is the value of I_3, not I_1.

Choice E (19.6% of cohort). The student made the same mistake as in choice D. But he/she may have attempted to “rectify” it by changing the sign.

43. Correct answer B (23.5% of cohort).

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$U = \frac{1}{2} C_{eq} (\Delta V)^2 = 0.09J$$



Choice A (17.9% of cohort). The student may have remembered the formula wrongly as $U = \frac{1}{2} C_{eq} \Delta V$

Choice C (27.5% of cohort). The student may have remembered the formula wrongly as $U = C_{eq} (\Delta V)^2$

Choice D (17.9% of cohort). The student may have remembered the formula wrongly as $C_{eq} = C_1 + C_2$

Choice E (10.2% of cohort). The student may have made both mistakes as in choices B and D.

44. Correct answer B (22.6% of cohort).The root-mean-square voltage across *MN* will not change because it only depends on the ratio of the number of coils of the primary loop to that of the secondary loop.

Choice A (13.9% of cohort). The equivalent resistance of L_1 and L_2 is less than the resistance of L_1. The total resistance of the secondary loop will be smaller. The total root-mean-square current flowing through the secondary loop will be larger. Therefore, the root-mean-square voltage across the resistance will be larger.

Choice C (20.8% of cohort). The voltage across L_1 will be smaller. So, the current will be smaller.

Choice D (16.5% of cohort). The root-mean-square current in the secondary loop is proportional to the root-mean-square current in the primary loop. If the current in the secondary loop becomes larger (see A), so does the current in the primary loop.

Choice E (24.2% of cohort). The total power of the secondary loop is $P = UI$. Here U is the root-mean-square voltage across *MN,* which has no change and I is the root-mean-square current in the secondary loop, which will be larger. So, the total power will be larger.

45. Correct answer E (8.6% of cohort). The circuit shown in the question cannot be realized using ideal elements only. If real batteries are used, we would need to take into account their internal resistances.

Choice A (40.6% of cohort). Students may have subtracted the ‘current’ due to the 3-V battery from that due to the 5-V battery.

Choice B (2.7% of cohort). Students may erroneously assume that the 3-V battery dictates the potential difference across the 10-Ω resistor.

Choice C (9.6% of cohort). Students may erroneously assume that the 5-V battery dictates the potential difference across the 10-Ω resistor.

Choice D (37.9% of cohort). Students may have added the ‘currents’ due to the 3-V and 5-V batteries.

46. Correct answer C (16.4% of cohort). Na^+ is lighter than Cl^-, and thus have a higher mobility. For the same potential difference between the two electrodes, the Na^+ ion current will be larger than the Cl^- ion current.

Choice A (50.1% of cohort). Students may have applied the erroneous logic that ions with the same charges will carry the same current.

Choice B (14.4% of cohort). Students may not realize that the current carried by the Cl^- ions is positive even though Cl^- ions move in the opposite direction to the Na^+ ions.

Choice D (10.6% of cohort). Students may recognize that ions with different masses may have different mobilities, but assign erroneously higher mobility to the Cl^- ions.

Choice E (8.1% of cohort). Students may have assumed erroneously that electric currents can only be carried by free electrons.

47. Correct answer D (66.8% of cohort). This answer can be calculated from summing a geometric series $R + R/2 + R/4 + R/8 + \ldots = R/(1 - 1/2) = 2R$., if we recognize that each stage of the infinite circuit is a parallel circuit of the previous stage.

Choice A (1.9% of cohort). Students may have considered successive stages in the infinite circuit, find their resistances to be decreasing, and apply the erroneous logic that the effective resistance of the circuit is that of its final stage.

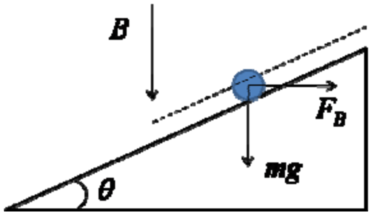
Choice B (10.6% of cohort). Students may have found the number of resistors increasing, and apply the erroneous logic that a circuit with infinitely many resistors will have infinite effective resistance.

Choice C (4.3% of cohort). Students may not recognize the series-parallel addition nature of the infinite circuit.

Choice E (16.1% of cohort). Students may not recognize the series-parallel addition nature of the infinite circuit.

48. Correct answer D (17.3% of cohort). This question tests the balance between gravitational force and magnetic force. The rod is released from rest, so its velocity will increase due to decrease in the gravitational potential. On the other hand, the increase in velocity also means the magnetic "dragging force" increases which is a result of Faraday's law (induced current due to change of magnetic flux). At the terminal velocity, the two forces along the slope should equal, that is, $mg\ \sin\theta = BIl\ \mathrm{con}\ \theta$. The key point is two things: First, the magnetic flux should be $BA\mathrm{con}\theta$, rather than just BA, since the loop's normal direction has an angle θ with that of the magnetic field. Second, the direction of the force. $I = \frac{\xi}{R} = \frac{1}{R}\frac{\Delta\Phi}{\Delta t} = \frac{B\ell v\cos\theta}{R}$. So the magnetic force $F_B = BI\ell = \frac{B^2\ell^2 v\cos\theta}{R}$.

Since at terminal velocity, $mg\sin\theta = BIl\mathrm{con}\theta$, we have $v_T = \frac{mgR}{B^2l^2}\frac{\sin\theta}{\mathrm{con}^2\theta}$



Choice A (8.1% of cohort). The student may not consider the balance of force along the slope. They simply use the most-commonly applied equation here.

Choice B (33.5% of cohort). The student considered the projection of gravitational force along the slope, but may take it for granted that the magnetic force is also in opposite direction of the velocity. Also, the student may not be aware of the angle θ when formulizing the magnetic flux.

Choice C (28.9% of cohort). The student considered the force balance along the slope, but miss one of $\cos\theta$ terms as mentioned above.

Choice E (8.0% of cohort). The student might choose this just by guessing.

49. Correct answer D (26.7% of cohort). This question tests the charge distribution of conductors at equilibrium condition. Before grounding, the inner surface of the hollow shell should have homogeneous distributed charges of –Q, while the outer

surface should have charges of +2Q–(–Q) = +3Q. When the shell is grounded, the charge on the outer surface is neutralized and thus zero. The charges on the inner surface remains the same which is stabilized by the +Q of the sphere.

Choice A (15.8% of cohort). The student may think the same type of charges repel each other, so that all the +2Q charge are pushed to the outer surface of the shell.

Choice B (23.7% of cohort). The student may think similar as above. After grounding, the inner surface should appear –Q to compensate the +Q on the sphere.

Choice C (20.4% of cohort). The student may think once a conductor is grounded, there should no charge anywhere on the conductor.

Choice E (12.2% of cohort). The student may just guess.

50. Correct answer is C (36.1.% of cohort). This question tests the definition of free charges on parallel-plate capacitor and the influence of dielectrics. Since the capacitor is always connected with the battery, its voltage is kept the same. After inserting the dielectrics, the capacitance increases since $C = \frac{\kappa \varepsilon_0 A}{d}$. According to the relationship $C = Q_{\text{free}} / \Delta V$, the free charge density σ_{free} should also increased by κ times. This means the electric field E_{free} due to free charges also increased by κ times. But according to the definition of dielectric $\kappa = E_{\text{free}} / E_{\text{total}}$, the total electric field between the plates remains the same, which is ΔV/d.

Choice A (9.9% of cohort). The student may be unfamiliar with the definition of capacitance.

Choice B (26.5% of cohort). The student may think the electric field should increases since the capacitance increases.

Choice D (18.6% of cohort). The student may think, since the electric field is induced by the free charges on the plates, an increase in the free charges should also correspond to an increase in the electric field. In this case, the influence of induced field is not considered.

Choice E (5.4% of cohort). The student may just guess.

51. Correct choice is B (27.9% of cohort). This is a relatively simple question which tests the Ampere's Law. The student should consider not only the magnitude but also the direction of the magnetic fields induced by the two currents. In order to be zero, the direction should be opposite. It's easy to see this by drawing the directions of the magnetic field in I, II, and III regions for both currents. Ignoring

the constant in the Ampere's law $B = \dfrac{\mu_0 I}{2\pi r}$, we can write $\dfrac{5.0}{r} - \dfrac{8.0}{r+12} = 0$, which gives $r = 20$ cm.

Choice A (20.7% of cohort). The student may be unsure about the direction of the magnetic fields.

Choice C (15.1% of cohort). The student knows the direction, but may think the magnitude is inversely proportionally to square of r (which is a common relationship in physics).

Choice D (31.9% of cohort). The student may use the correct equation for calculation but not notice the direction. $\dfrac{5.0}{r} - \dfrac{8.0}{12-r} = 0$ which gives r ≈ 4.6 cm.

Choice E (3.0% of cohort). Wrong calculation.

52. Correct choice is E (24.8% of cohort). Using the Lorentz force, electron carriers will experience a force $F = evB$ downwards, leading to an emf of $|\varepsilon| = Blv$ between the two bar-loop contact points, where v is the speed of the bar, B is the magnetic field, and l is the length of the bar fragment (or distance) between the two bar-loop contact points

Hence the current along the metal bar is $I = \varepsilon/R = Blv/R$, where R is the resistance of the bar between the two bar-loop contact points.

For the uniform metal, the resistance of a bar fragment (between two contact points) of the length l can be expressed as $R = \rho l$, where ρ is the resistance per bar length unit.

In conclusion, the current in the bar is: $I = Blv/\rho l = Bv/\rho$, which is independent of the distance between the two bar-loop contact points or the shape of the loops. Therefore, $I_R = I_T = I_E$

Choice A (4.9% of cohort). The student makes a guess.

Choice B (47.1% of cohort). The student may consider the difference in emf, but not the difference in resistance.

Choice C (17.5% of cohort). The student may consider the difference in resistance, but not the difference in emf.

Choice D (4.3% of cohort). The student makes a guess.

53. Correct choice is C (31.7% of cohort). The mobile free electrons will accumulate at the bottom portion of the plate due to Lorentz force ($\vec{F_B} = -e\vec{v}\times\vec{B}$) acting on it. Subsequently the top portion will have a deficit of electrons (positively charged). This distribution will create an electric field pointing downwards. It will continue until the electrostatic force produced by this field is balanced with the magnetic force.

$$F_{el} = F_B$$

$$eE = e\frac{\Delta V}{l} = evB$$

$$\Delta V = Blv$$

Since the electrons accumulate at the bottom, V_2-V_1 has to be negative, hence V_2-$V_1 = -Blv$

Choice A (24.4% of cohort). The student may have applied the left hand rule incorrectly, or may have forgotten the negative sign on the electron charge.

Choice B (12.1% of cohort). Random guess, or the student did not understand the question.

Choice D (11.3% of cohort). Random guess, or the student did not understand the question.

Choice E (18.7% of cohort). Random guess, or the student did not understand the question.

54. Correct choice is C (23.1% of cohort). When the current flows from A to B, the flux due to this current will be in downward direction (using right hand rule). The increase of the current means that the flux that enters the coil will increase as well. Thus there will be an induced emf such that the the direction is against the direction of the increase of the flux (Lenz's law). In this case, A will be more positive than B.

When the current flows from B to A, the flux due to this current will be in upward direction (using right hand rule). The increase of the current means that the flux that enters the coil will increase as well. Thus there will be an induced emf such that the the direction is against the direction of the increase of the flux (Lenz's law). In this case, A will be more positive than B.

Choice A (48.9% of cohort). Students may have thought that this is similar to the case with resistor, where if the current flows from A to B, the potential at A is then higher than potential at B.

Choice B (12.9% of cohort). Random guess.

Choice D (8.4% of cohort). Students may have failed to identify the orientation of the coil (clockwise or anticlockwise) and therefore results in wrong answer since the orientation of the coil is important to determine the direction of the flux due to the current (right hand rule).

Choice E (4.5% of cohort). 1 to 2 is correct. 3 to 4 may have been the result of the students' failure to identify the orientation of the coil (clockwise or anti-clockwise) and therefore results in wrong answer since the orientation of the coil is important to determine the direction of the flux due to the current (right hand rule).

55. Correct choice is E (22.2% of cohort). (i) is correct because the transformer steps down the input voltage by a factor of 10. (ii) is correct because the ammeter measures $I_{\text{rms}} = 1$ A, and from Figure 1, we see that $V_{\text{rms}} = V_{\text{max}}/\sqrt{2} = 200$ V. Thus $\overline{P} = V_{rms}I_{rms} = (200)(1) = 200$ W. (iii) is correct because $T = 0.02$ s from Figure, hence $f = 1/T = 50$ Hz.

Choice A (3.7% of cohort). Students fail to realize that (i), (ii), and (iii) are all correct.

Choice B (22.0% of cohort). Students fail to realize that (i), (ii), and (iii) are all correct.

Choice C (41.1% of cohort). Students fail to realize that (i), (ii), and (iii) are all correct.

Choice D (7.6% of cohort). Students worried that (iv) might be correct.

56. Correct choice is C (44.7% of cohort). From the figure, we see that the potential difference across *ab* always has the same polarity as the potential difference across *cd*. Therefore, like connecting two batteries in series, we need to connect *b* and *c*, and use *a* and *d* for output, which is option (i), or we can connect *a* and *d*, and use *b* and *c* for output, which is option (ii).

If we connect *a* and *c*, and use *b* and *d* for output, or connect *b* and *d*, and use *a* and *c* for output, it would be like connecting two batteries in parallel. In these cases, the output voltages would depend on the resistances of the two secondary circuits, but neither would be 8 V.

Choice A (30.7% of cohort). Students see the logic for case (i), but failed to recognize that the same logic applies to case (ii).

Choice B (4.7% of cohort). Students arrive at cases (iii) or (iv) using a flawed logic.

Choice D (8.8% of cohort). Students arrive at cases (iii) or (iv) using a flawed logic.

Choice E (8.4% of cohort). Students feel that the question is more tricky than it really is.

57. Correct choice is C (23.3% of cohort). The peak value of u_2 is $50\sqrt{2}$ V. Since we are dealing with a half-wave rectifier, the rms voltage that develops across R_L would be half the peak value of u_2., i.e. $u_{0,rms} = u_{2,\max}/2 = 25\sqrt{2}$ V. However, the question wants the *average* voltage across R_L, and this is $\overline{u}_0 = u_{2,\max}/\pi = 22.5$ V. The average power that develops across R_L is $\overline{P} = u_{0,rms}^2 / R_L = 1250/250 = 5$ W.

Choice A (19.2% of cohort). Students do not understand how the half-wave rectifier works, and are unsure how to calculate the average power.

Choice B (32.6% of cohort). Students understands that the rms value of a half-wave rectified voltage is half the peak value, but misread 50 V as the peak value, and at the same time are unsure how to calculate the average power.

Choice D (10.3% of cohort). Students do not understand how the half-wave rectifier works, and are unsure how to calculate the average power.

Choice E (10.7% of cohort). Students understands how the half-wave rectifier works, and can calculate the average DC voltage across the load resistor, but are unsure how to calculate the average power.

Modern Physics

58. Correct answer D (37.4% of cohort). $\Delta V = 0.003\%$ x 5 x $10^3 = 0.15\text{ms}^{-1}$
$\Delta P = M\Delta V = (9.11 \text{ x } 10^{-31} \text{ x } 0.15) = 1.3665 \text{ x } 10^{-31}\text{kgms}^{-1}$

From Heisenberg Uncertainty,

$$\Delta x \Delta P \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta x \geq \frac{\hbar}{2\Delta P}$$

$$\geq 0.384mm$$

Choice A (12.0% of cohort). The student makes a guess.

Choice B (18.5% of cohort). The student makes a guess.

Choice C (20.7% of cohort). The student makes a guess.

Choice E (6.8% of cohort). The student makes a guess.

59. Correct answer D (17.9% of cohort). The statistics for a thermal light source and a laser light source are different.

Choice A (13.1% of cohort). The student wrongly thought that laser light has only particular wavelengths. However thermal light and laser light can have the same wavelength.

Choice B (12.9% of cohort). Thermal light and laser light can have the same wavelength and the linewidth of thermal light sources can be narrowed down with filters to make them comparable to laser light sources.

Choice C (45.1% of cohort). The student may think that the laser light always has to be collimated which is of course not true since the beam divergence determined by the resonator and can be the same as for a thermal light beam.

Choice E (7.4% of cohort). The student makes a guess.

60. Correct answer C (41.6% of cohort).

Initially,

$$\frac{N_{14}(0)}{N_{12}(0)} = R = 1.3 \text{ x } 10^{-12}.$$

After a time T, we have

$$N_{14}(T) = N_{14}(0)e^{-\lambda T}$$
$$N_{12}(T) = N_{12}(0) + N_{14}(0)\left(1-e^{-\lambda T}\right) = N_{14}(0)\left(1/R+1-e^{-\lambda T}\right)$$
$$\Rightarrow \frac{N_{14}(T)}{N_{12}(T)} = \frac{e^{-\lambda T}}{\left(1/R+1-e^{-\lambda T}\right)} \qquad (1)$$

where λ is the rate of ^{14}C decay, which can be deduced from the ^{14}C half-life by:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{(5730\ yr)(3.16\times10^{7}\ s/yr)} = 3.83\ \times 10^{-12}\ s^{-1}$$

From the date given we can determine $\frac{N_{14}(T)}{N_{12}(T)}$ as follow:

$$\frac{dN_{14}(t)}{dt} = -\lambda N_{14}(t)$$

$$\Rightarrow N_{14}\,(T) = \left(\frac{195}{60}\right)\left(\frac{1}{3.83\ \times\ 10^{-12}}\right) = \ 0.8486\ \times\ 10^{12}$$

$$N_{12}(T) = \left(25g - \frac{N_{14}(T)}{N_A}14g\right)\left(\frac{1}{12g}\right)N_A$$

$$= 1.25417 \times\ 10^{24}$$

$$\Rightarrow \frac{N_{14}\,(T)}{N_{12}\,(T)} = \frac{0.8486\times\ 10^{12}}{1.25417\times\ 10^{24}} = \ 0.6766\ \times\ 10^{-12} \qquad (2)$$

(1) and (2) give

$$0.6766\ \ \mathrm{x}\ 10^{-12} = \frac{e^{-\lambda T}}{\left(1/1.3\ \mathrm{x}\ 10^{-12}+1-e^{-\lambda T}\right)} \approx 1.3\ \mathrm{x}\ 10^{-12}e^{-\lambda T}$$

$$\Rightarrow T\ = \ \frac{-1}{\lambda}\ln = \left(\frac{0.6766\ \times\ 10^{-12}}{1.3\ \times\ 10^{-12}}\right)$$

:. $T \approx$ 5,400 years

Choice A (4.0% of cohort). The student makes a guess.

Choice B (9.0% of cohort). The student makes a guess.

Choice D (33.1% of cohort). The student makes a guess.

Choice E (7.0% of cohort). The student makes a guess.

Note: Percentages based on Singapore Cohort