Interior-Point Methods in Convex Optimization

Chua Chek Beng
Division of Mathematical Sciences, School of Physical and Mathematical Sciences
Nanyang Technological University, Singapore 637371

Optimization is a major tool in various scientific, engineering and financial disciplines. As its name suggests, optimization tries to make optimal use of a set of resources to achieve a set of objectives within the given constraints of a situation. An optimization algorithm might, for example, show how to manufacture artificial blood, given multiple chemical components, each with different handling requirements, costs, and chemical properties, within a given time. With direct applications in computer science, engineering, physical and management sciences, and mathematics, optimization has grown enormously in recent years as the importance of optimizing approaches to high technology applications have become understood. For instance, in designing a bridge, semidefinite programming can be used to solve a truss design problem that aims to minimize compliance of the bridge so that it is most resistant to external force.

Semidefinite optimization is a widely used mathematical model for numerous real-world problems such as control system design, VLSI circuit design, truss topology design, portfolio optimization, option pricing, etc. These structured convex optimization models are also used in approximating very hard combinatorial problems. This is best exemplified by the Goemans and Williamson's algorithm for approximating the maximum cut of a graph. A semidefinite optimization model was used in this algorithm to approximate the value of the maximum cut.

We study the theory of convex optimization, and the application of interior-point method in solving convex optimization models. Interior-point method is a class of algorithms that searches the interior of the set of feasible solutions for the optimal solution. It has been used extensively to provide efficient algorithms for many classes of optimization models, including linear optimization, convex quadratic optimization and semidefinite optimization. We have designed and analyzed efficient algorithms for various structured convex optimization problems such as target-following algorithms for semidefinite optimization and symmetric cone optimization, and primal-dual interior-point algorithms for homogeneous cone optimization.

A fundamental object in the theory of interior-point method is the primal-dual central path. Path-following algorithms—an important sub-class of interior-point algorithms—search for optimal solutions by following central paths. In one of our project, we investigated and proved several properties of the primal-dual central paths for semidefinite optimization and homogeneous cone optimization. These properties are useful in the study of local convergence behaviour of path-following algorithms. For instance, these results explain the local super-linear convergence of primal-dual interior-point methods for semidefinite optimization problems.

Selected Publications:

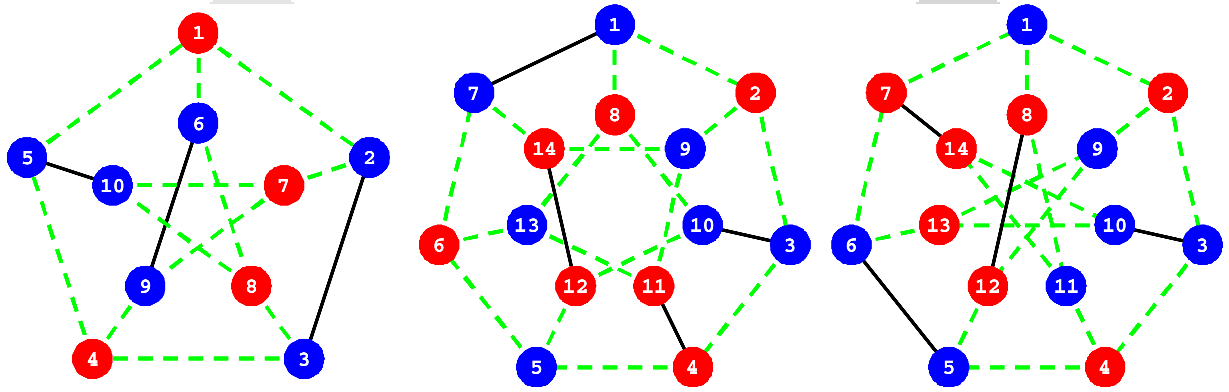
Chua, C.B., Relating homogeneous cones and positive definite cones via T-algebras, SIAM J. Optm. 14:500-506 (2003)

Chua, C.B., A new notion of weighted centers for semidefinite programming, SIAM J. Optm. 16:1092-1109 (2006)

Chua, C.B., The primal-dual second-order cone approximations algorithm for symmetric cone programming, Found. Comput. Math. 7: 271-302 (2007)

Chua, C.B. and Tunçel, L., Invariance and efficiency of convex representations, Math. Prog.. 111: 114-140 (2008)

Chua, C.B., Analyticity of weighted central path and error bound for semidefinite programming, Math. Prog., 115: 239-271 (2008)



These are examples of max-cut problems. The first graph is known as Peterson's graph. It is often used as a counter-example to conjectures in graph theory. The next two are examples of one type of generalization of Peterson's graph. For these examples, the semidefinite optimization models are tight --- solving the models gives the maximum cuts for these graphs. In each of these graphs, the nodes are grouped according to their colour. The green dashed arcs are those that contribute to the cut.