

Random Matrix Theory: From Theoretical Studies to Applications

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Random Matrices

Random matrix theory is a paradigm for describing and understanding a variety of phenomena in many disciplines. The theory dates back to Wigner's work on nuclear physics in the 1950's. In quantum mechanics, the energy levels of a system are described by the eigenvalues of a Hermitian operator on a Hilbert space. Hence physicists working on quantum mechanics are interested in the asymptotic behavior of eigenvalues and from then on, random matrix theory becomes a very popular topic among mathematicians, probabilists and statisticians.

So far in statistics, this line of work has been concerned mostly with the asymptotic properties of sample covariance matrices. Since the M-P law was discovered, there have been accumulated fruitful results regarding eigenvalues of sample covariance. However, in contrast with eigenvalues, eigenvectors are not well understood. It is well known that the eigenvector-matrix of the Wishart matrix has the Haar distribution, that is, the uniform distribution over the group of unitary matrices (or orthogonal matrices for the real case). It is conceivable that the eigenvector-matrix of large sample covariance matrices should be "asymptotically Haar distributed" or at least in some way close to the Haar measure. Toward this aim, we introduce an empirical distribution involving eigenvalues and eigenvectors of generalized sample covariance matrices and prove that it has the same limit as the classic one. In addition, we also establish central limit theorems of eigenvalues under more general assumption (without kurtosis three).

Previously, in wireless communications, in order to analyze system dependent parameters, researchers relied mainly on exhaustive simulations to determine the performance of communication systems. But, by using random signature sequence models, random matrices have proven useful in performance analysis and system design. We have developed the asymptotic properties of Signal-to-Interference ratio (SIR) of a suboptimal receiver and principal component receiver. Moreover, the asymptotic distribution of Sum Mutual Information of linear receivers in DS-CDMA or MIMO systems is also characterized.

Publications

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