New Lower Bound Methods in Communication and Query Complexity

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In the model of communication complexity one studies the amount of communication that is necessary to solve problems with inputs distributed to two or more parties. Originally introduced by Yao in 1979, results about this model have found a large number of applications to other computational problems, e.g., in circuit complexity, time-space tradeoffs, datastream algorithms, VLSI circuits, data structures, auction theory, and even the limits of current techniques to prove lower bounds for Turing machines. Besides these applications, in which usually lower bounds for specific communication tasks are used to bound resources in the aforementioned settings, the communication model is also interesting to study in its own right, as a setting, in which different modes of computation, such as determinism, nondeterminism, randomization can be compared and investigated.

The model of query complexity is possibly the simplest computational model of interest and studies the number of input positions that need to be read before a function can be computed.

In recent work (with Rahul Jain) we have introduced a family of methods to prove lower bounds in both models, based on linear programming formulations. A key idea in this approach is that computations lead to a decomposition of the set of inputs into simpler objects (called rectangles resp. subcubes), which form a partition. In contrast previous techniques do not use that e.g. rectangles induced by a communication protocol do not intersect. As a result we were able to describe the strongest known general lower bound techniques for randomized communication protocols and query algorithms.

In particular the new techniques allowed proving a so-called strong direct product theorem for the disjointness problem in communication complexity. This problem is probably the most studied problem in the area. We have shown that in order to solve $k$ instances of the disjointness problem on independent inputs by a randomized protocol, either the communication has to be almost the maximum possible, or the success probability is exponentially small in $k$ (and hence not much better than giving a random output). Interestingly, the corresponding quantum communication complexity had been settled much earlier in a joint paper with Spalek and de Wolf.

An interesting corollary of this result is the following tradeoff. If two players Alice and Bob receive an $n \times n$ Boolean matrix each, and want to compute the Boolean matrix product, then the product of the workspace $S$ of each player (excluding the inputs) and the communication $C$ must be at least $CS = \Omega(n^3)$. This is the optimal lower bound, since a matching upper bound is easy to see for all space bounds $S$ between $\log n$ and $n$.

Publications


Dr. Klauck is an Assistant Professor in the Division of Mathematical Sciences. He received his PhD from the University of Frankfurt, held postdoctoral positions at CWI (Amsterdam), the Institute for Advanced Study (Princeton) and the University of Calgary before joining the University of Frankfurt as a Junior Research Group Leader. He joined NTU in April 2010 and is also a Principal Investigator at the Centre for Quantum Technologies in Singapore.