

Sparse Tensor Approximation of Elliptic sPDEs

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Date : 13 February 2009 (Friday)
Time : 3.30 pm – 4.30 pm
Venue: SPMS-Executive Classroom 1, MAS-03-06
School of Physical and Mathematical
Sciences

We consider the Finite Element Solution of second order elliptic problems in a physical domain $D \subset \mathbb{R}^d$ with spatially inhomogeneous random coefficients. We present convergence rates and complexity estimates for deterministic, sparse Galerkin semidiscretization in the probability domain of the random solution. It is parametrized in polynomials of the first M Karh´unen-Lo`eve (KL) variables of the input data [TS06]. Two cases are distinguished: (i) Exponential decay of the input’s KL expansion based on [TS] and (ii) algebraic decay of the input’s KL expansion. In (i), a “polynomial chaos” type Galerkin discretization is shown to yield spectral convergence rates in terms of $N\Omega$, the number of deterministic elliptic problems to be solved. In (ii), approximation rates in terms of $N\Omega$ are higher than rates obtained for Monte-Carlo. Finally, in (iii) ongoing work [BS, ABS08, CDS09] on the total complexity vs. accuracy of (adaptive) tensor Galerkin discretizations in both, stochastic as well as in the deterministic domain D will be addressed. Sufficient conditions on the joint pdf’s of the random field input to ensure better complexity than with (Quasi) Monte Carlo in the probability domain and with Galerkin discretization in D will be identified and implementational issues will be addressed.

References:

[TS06] Ch. Schwab and R.A. Todor, Karh´unen-Lo`eve Approximation of Random Fields by Generalized Fast Multipole Methods. *Journal of Computational Physics* 217 (2006), 100-122.

[TS] R.A. Todor and Ch. Schwab, Convergence Rates of Sparse Chaos Approximations of Elliptic Problems with stochastic coefficients. *IMA Journ. Numer. Anal.* (2007). [BS] M. Bieri and Ch. Schwab, Sparse high order FEM for elliptics PDEs (in press in *Comp. Meth. Appl. Mech. Engg.* 2009). [ABS08] M. Bieri, R. Andreev and Ch. Schwab, Sparse Tensor Galerkin Approximations for elliptic sPDEs (in review 2009). [CDS09] A. Cohen, R. DeVore and Ch. Schwab Convergence Rates of best N -term approximations for a class of elliptic sPDEs (in review 2009).

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Speaker Biography

Christoph Schwab is a Professor in the Department of Mathematics at the Swiss Institute of Technology (Zurich). He obtained his PhD from the University of Maryland, College Park. Professor Schwab is a leading expert in Numerical Analysis. He is, or has been, a member of the Editorial board of many leading numerical analysis and applied mathematics journals. He has been a plenary and invited speaker at many international conferences, most notably, at the Numerical Analysis section of the International Congress of Mathematicians 2002 in Beijing. His research interests include numerical solutions of partial differential equations with stochastic data, computational methods for quantitative finance, problems with multiple scales and wavelet- and hp-methods.

Host: Prof. Hoang Viet Ha, Division of Mathematical Sciences, School of Physical and Mathematical Sciences

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