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Mock theta functions

The mock theta functions, possibly the most famous and the deepest topic in Ramanujan's lost notebook, were first introduced by Ramanujan in his last letter to G. H. Hardy. In his letter, he called a function $f(q)$ defined by a power series in q which converges for $|q|<1$ a mock theta function if it satisfies

(1) For every root of unity ζ, there is a θ-function $\theta_\zeta(q)$ such that the difference $f(q)-\theta_\zeta(q)$ is bounded as $q \to \zeta$ radially.

(2) There is no single θ-function which works for all ζ; i.e., for every θ-function $\theta(q)$ there is some root of unity ζ for which $f(q)-\theta(q)$ is unbounded as $q \to \zeta$ radially.

Ramanaujan also gave a list of 17 functions as examples of mock theta functions, classified according to their respective orders. Unfortunately he did not explain what he meant by the "order".

Since the works of many mathematicians, which include G .E. Andrews, K. Bringmann, Y.-S. Choi, F. G. Garvan, B. Gordon, D. Hickerson, R. J. Mcintosh, K. Ono, G. N. Watson, and S. P. Zwegers, many doubts about Ramanujan's mock theta functions have been cleared. However, there remains, much to be discovered and understood about the mock theta functions. Together with B. C. Berndt, we discovered two new mock theta functions. Further exploration on the properties of mock theta functions is being carried out.

Problems related to the partition function, *p*(*n*).

The partition function, $p(n)$, which counts the number of ways of representing an integer n as a sum of nonincreasing positive integers, was first studied by Euler. Since the discovery of the three congruences by Ramanujan,

$$p(5n+4) \equiv 0 \pmod 5,$$
$$p(7n+5) \equiv 0 \pmod 7,$$
$$p(11n+6) \equiv 0 \pmod{11},$$

the partition function became an object of study for many mathematicians.

In attempting to find combinatorial interpretations for these congruences, various generalizations of the partition function were introduced. In particular, in 1944, F. J. Dyson introduced the *rank of a partition* and conjectured the existence of another partition statistic *crank*. The *crank* was discovered by Andrews and Garvan in 1986.

Various properties on the ranks and the cranks are awaiting to be uncovered and proved. Modular forms, theta functions, and Lambert series are some useful tools in this topic.

Selected Publications

B. C. Berndt, H. H. Chan, S. H. Chan, and W.-C. Liaw, Cranks and dissections in Ramanujan's lost notebook, J. Combin. Theory Ser. A, 109 (2005), no.1, 91—120.

B. C. Berndt and S. H. Chan, Sixth order mock theta functions, Adv. Math., 216 (2007), no.2, 771—786.

B. C. Berndt, S. H. Chan, Z.-G. Liu, and H. Yesilyurt, A new identity for $(q;q)^{10}_\infty$ with an application to Ramanujan's partition congruence modulo 11, Quart. J. Math, 55 (2004), no. 1, 13—30.

S. H. Chan, A short proof of Ramanujan's famous ${}_1\psi_1$ summation formula, J. Approx. Theory, 132 (2005), no.1, 149—153.

S. H. Chan, Generalized Lambert series, Proc. London Math. Soc., 91 (2005) no.3, 598—622.